Enroll No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER – 2016

16PMTCC02 - TOPOLOGY - I

Duration of Exam – 3 hrs ***Semester – I*** ***Max. Marks – 70***

***Part A** (5x2= 10 marks)*

Answer **ALL** questions

1. If X ={1, 2, 3, 4, 5}and 𝓣 = {Ø, X, {1, 2, 3},{2, 3, 5},{1, 5}}then verify weather (X, 𝓣) is a topological space or not..
2. Define: (i) Order Topology (ii) Dictionary Order Relation.
3. If $\mathbb{Z}_+$ is the set of all the positive integers then $\mathbb{Z}_+' = ?$ and $\overline{\mathbb{Z}_+} = ?$ Justify.?
4. Define: (i) Continuous Function (ii) Homeomorphism (w. r. t topological spaces)
5. Define: (i) T_3 - space (ii) T_4 - space.

***Part B** (5X5 = 25 marks)*

Answer **ALL** questions

6a. Define: (i) Topology.
(ii) Standard topology on R.
Let X be a set, Let be a basis for a topology ² on X. Then prove that ² equals the collection of all unions of elements of .

OR

6b. *If* {²~ / ~ ∈ *I*} *is a colloction of topologies on X.*
then show that $\bigcap_{\sim \in I}$ ²~ *is a topology on X.*
Is $\bigcup_{\sim \in I}$ ²~ *a topology on X? Justify your answer?*

7a. Let (X, 𝓣) be a topological space. Let Y ⊂ X, B be a basis for topology 𝓣 *of X* then prove that the collection $\mathcal{B}_Y$ = {B∩Y / B∈ 𝓑} is a basis for the subspace topology $\mathcal{T}_Y$ on the set Y.

OR

7b. Define: Open Rays and Closed Rays.
Prove that every order topology is a Hausdorff topology .

8a. Define: Neighbourhood of a point.
Let A be a subset of the topological space X, then x ∈ $\overline{A}$ if and only if every open set U containing x intersects A.

OR

8b. Let (X, $\mathcal{T}$) be a topological space with topology $\mathcal{T}$ on it and let $\mathcal{B}$ be a basis for the topology $\mathcal{T}$ on X and let A ⊂ X then prove that $x \in \overline{A} \Leftrightarrow$ for every $B \in \mathcal{B}$ with $x \in B$, $B \cap A \neq \varnothing$.

9a Let f (X, d) be a metric space, let $B_d(x, \varepsilon) = \{y \in X / d(x,y) < \varepsilon\}$ denote the ε - ball at the point x ϵ X, Let $\mathcal{B} = \{ B_d(x, \varepsilon) / x \in X, \varepsilon > 0 \}$ then prove that $\mathcal{B}$ is a basis for a topology.

OR

9b. Let (X, $\mathcal{T}_X$) and (Y, $\mathcal{T}_Y$) be topological spaces, Let f: (X, $\mathcal{T}_X$)→ (Y, $\mathcal{T}_Y$) be a function of topological spaces, then prove that f is continuous function if and only if for each x ∈ X and each neighbourhood V of f(x), there is a neighbourhood U of x such that f(U) ⊂ V.

10a. Define: Hausdorff Space
Prove that a subspace of T_2-space is a T_2-space.

OR

10b. Define: T_1 - space.
If X is a Hausdorff space then prove that a sequence of points of X converges to at most one point of X.

Part C ***(5X7 = 35 marks)***

Answer **ALL** questions

11a. Let (X, $\mathcal{T}$) be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and for each x ∈ U there is an element C of $\mathcal{C}$ such that x ∈ C ⊂ U, then **prove that** $\mathcal{C}$ is a basis for the topology of X.

OR

11b. Let $\mathcal{B}$ and $\mathcal{B}'$ be basis for the topologies of $\mathcal{T}$ and $\mathcal{T}'$ respectively on X, then **prove that** $\mathcal{T} \subset \mathcal{T}'$ implies for each x ∈ X and each basis element B ∈ $\mathcal{B}$ containing x, there is a basis element B' ∈ $\mathcal{B}'$ such that x ∈ B' ⊂ B.

12a (i) Explain and discuss the order topology on the set $\mathbf{Z}^+$.
(ii) Let (X, $\mathcal{T}$) be a topological space, and Y be a subspace of X, if V ⊂ Y is open in Y and Y is open in X then prove that V is open in X.

OR

12b. Define: Simple Order Relation.
If (X, $\mathcal{T}$) is a topological space, If Y is a subset of X then prove that the collection $\mathcal{T}_Y = \{Y \cap U / U \in \mathcal{T}\}$ is a topology.

13a. Define: Closure of a Set (of a topological space).

Let (X, $\mathcal{T}_1$) and (Y, $\mathcal{T}_2$) be topological spaces. What is the sub-basis for the product topology on X×Y, Justify your answer with proof.

OR

13b. Define: Accumulation Point (of a subset of a topological space)

Let (X, $\mathcal{T}$) be a topological space with topology T on it, then **prove that** the following conditions holds: (i) ∅ and X are closed sets. (ii) Arbitrary intersection of closed sets is closed. (iii) Finite union of closed sets is closed.

14a. Define: Metrizable Topological Space.

Let (X, $\mathcal{T}_X$) and (Y, $\mathcal{T}_Y$) be topological spaces, Let f: (X, $\mathcal{T}_X$)→ (Y, $\mathcal{T}_Y$) be a function of topological spaces, then prove that if for every closed set B of Y the set f^{-1}(B) is a closed set in X then f is continuous function.

OR

14b. Define: Topological Imbedding

. (i) Prove that the composition of two continuous functions is a continuous function.

(ii) Let (X, $\mathcal{T}_X$) and (Y, $\mathcal{T}_Y$) be topological spaces, Let f: (X, $\mathcal{T}_X$)→ (Y, $\mathcal{T}_Y$) be a continuous function of topological spaces, and let A be a subspace of (X, $\mathcal{T}_X$) then prove that the restricted function $f|_A$: A → (Y, $\mathcal{T}_Y$) is continuous.

15a. (i) Define: T_0-Space.

(ii) State the Urysohn Lemma.

Prove that every T_2-Space is T_1-Space.

Give an example of a topological space which is T_0-Space but not a T_1-Space.

OR

15b. (i) Prove that every finite point set in a Hausdorff space is closed.

(ii) Prove that every Metrizable space is normal.